

Problem 4

Suppose a metal rod laterally insulated has an initial temperature of 20°C but immediately thereafter has one end fixed at 50°C . The rest of the rod is immersed in a liquid solution of temperature 30°C . What would be the IBVP that describes this problem?

Solution

Suppose that L is the rod's length, that α^2 is the diffusivity constant, that k is the thermal conductivity, and that h is the heat exchange coefficient.

$$\begin{array}{ll} \text{PDE} & u_t = \alpha^2 u_{xx} \quad 0 < x < L \quad 0 < t < \infty \\ \text{BCs} & \begin{cases} u(0, t) = 50 \\ u_x(L, t) = -\frac{h}{k}[u(L, t) - 30] \end{cases} \quad 0 < t < \infty \\ \text{IC} & u(x, 0) = 20^\circ\text{C} \quad 0 \leq x \leq L \end{array}$$

Here it's assumed that the end at 50°C is at $x = 0$ and that heat flows from the ambient liquid at 30°C to the end at $x = L$.